Hanoi Open Mathematics Competition
Team Contest - Senior Section
Time limit: 60 minutes
Sample Questions

Information:

- You are allowed 60 minutes to complete 10 questions. For Questions 1, 2, 3, 4, 5, 6, 7, and 8, only numerical answers are required. For Questions 9 and 10, full solutions are required.
- Each one of Questions 1, 2, 3, and 4 is worth 5 points, and each one of Questions 5, 6, 7, and 8 is worth 10 points. No partial credits are given, and there are no penalties for incorrect answers. Each one of Questions 9 and 10 is worth 20 points, and partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your team's name in the space provided on the first page.
- Enter your answers in the space provided below the question.
- All together may discuss and complete the questions.
- The instruments such as protractors, calculators and electronic devices are not allowed to use.
- At the end of the contest you must put the question papers in the envelope provided.

- Write down your team's name in the space provided on every question sheet.

Team: _____ **Score:** _____

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No.	Questions										Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10		
Score												

Question 1. How many 3-digit numbers $\overline{abc}$ are there such that

$$a \times b \times c + a \times b + b \times c + c \times a + a + b + c = 29.$$

Answer: _____

Question 2: Find all 3-digit numbers $\overline{abc}$ such that $\overline{abc} = a! + b! + c!$.

Answer: _____

Question 3: The quadrilateral ABCD with area of $55\,cm^2$ and a point M in its interior such that

$MA = 3\,cm, MB = 4\,cm, MC = 8\,cm, MD = 6\,cm$. Find the perimeter of ABCD.

Answer: _____

Question 4: With usual notation in a ΔABC

$$\left(\frac{1}{r_A} + \frac{1}{r_B}\right)\left(\frac{1}{r_B} + \frac{1}{r_C}\right)\left(\frac{1}{r_C} + \frac{1}{r_A}\right) = \frac{kR^3}{a^2b^2c^2},$$

$\left(r_A, r_B, r_C \ are \ radius \ of \ Ex-circle\right)$, where 'k' has the value equal to

Answer: _____

Question 5. Let ABCD be a cyclic quadrilateral with $AB = 6,\ BC = 12,\ CD = 3\ and\ DA = 6$. Let E; F be the intersection of lines AB and CD, lines AD and BC respectively. Find EF

Answer: _____

Question 6. A regular polygon with m sides is drawn in the interior of another larger regular polygon with n sides such that they have one common side. If the measure of one angle of the larger polygon is 27^0 more than an angle of the other and $m + n = 28$.

Find the value of $S = 100m + 2n + 2$

Answer: _____

Question 7. Let a, b, c are real numbers such that

$$\begin{cases} a + 4b + 9c + 16d = 20 \\ 4a + 9b + 16c + 25d = 201 \\ 9a + 16b + 25c + 36d = 2017 \end{cases}$$

What is the value of $S = 16a + 25b + 36c + 49d$.

Answer: _____

Question 8. A student uses $10,000 to make a rectangle banner for his school. The length and width, in meter, of the banner are positive integer numbers. Each meter of length costs $330 and each meter of width costs $450. When the banner has largest area, what's perimeter of the banner, in meter?

Answer: _____

Question 9. Given three circles tangent to each other and to a straight line. If the radius of left circle is 4 cm and the right is 16 cm. Find the radius middle circle?
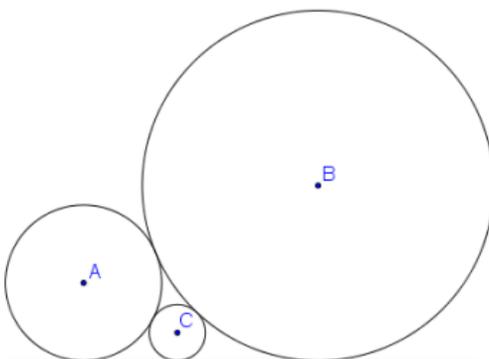
Solution:

Answer: _____

Question 10. Given positive real numbers x, y, z satisfying $x^2 + y^2 + z^2 = 1$.

Prove that

$$\frac{1}{1-xy} + \frac{1}{1-yz} + \frac{1}{1-zx} \leq \frac{9}{2}.$$

**Solution:**

Answer: _____