HMS
Hanoi Mathematical Society

# Hanoi Open Mathematics Competition
# Individual Contest - Junior Section
# Time limit: 120 minutes
# Sample Questions

## Information:

- You are allowed 120 minutes to complete 10 questions in Section A to which only numerical answers are required, and 5 questions in Section B to which full solutions are required.
- Each one of Questions1, 2, 3, 4, and 5 is worth 5 points, and each one of Questions 6, 7, 8, 9, and 10 is worth 10 points. No partial credits are given, and there are no penalties for incorrect answers. Each one of Question 11, 12, 13, 14, and 15 is worth 15 points, and partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

## Instructions:

- Write down your name, your contestant number and your team's name in the space provided on the first page of the question paper.
- For Section A, enter your short answers in the provided space. For Section B, write down your full solutions.
- You must use either pencil or ball-point pen which is either black or blue.
- The instruments such as protractors, calculators and electronic devices are not allowed to use.
- At the end of the contest you must put the question papers in the envelope provided.

**Team:** ________________ **Name:** __________________________ **No.:**_______ **Score:** _______

## For Juries Use Only

| No. | Section A | | | | | | | | | | Section B | | | | | Total | Sign by Jury |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | | |
| Score | | | | | | | | | | | | | | | | | |

## Section A.

**There are 10 questions. Fill in your answer in the space provided at the end of each question.**

**Question 1:** Calculate $\dfrac{\left[2017^2-\left(2015^2-\left(2013^2-\left(2011^2-...\left(3^2-1^2\right)\right)\right)\right)\right]-1}{1008}$

**A.** 2016. **B.** 2017. **C.** 2018. **D.** 2019. **E.** 2020.

**Question 2:** Let ΔABC be a triangle in which $\angle A=90^0, AB=AC, BC=36\,cm$. Draw rectangle MNPQ that $M\in AB$, $Q\in AC, N\in BC, P\in BC$. The possibly largest area of rectangle MNPQ is

**A.** 144. **B.** 169. **C.** 162. **D.** 146. **E.** 164.

**Question 3:** The last two digits of the number $2^{2016}+3^{2017}+5^{2018}$. is

**A.** 12. **B.** 02. **C.** 04. **D.** 24. **E.** 16.

**Question 4:** The function $f(x)$ has the following properties: $f\left(1\right)=1;\ f\left(2x\right)=4f\left(x\right)+6;$ $f\left(x+2\right)=f\left(x\right)+12x+12$. Find $f\left(6\right)$.

**A.** 10. **B.** 100. **C.** 46. **D.** 106. **E.** 10.

**Question 5:** Given real numbers $a,b$ satisfying

$$\begin{cases}a^3-3ab^2=26\\ b^3-3a^2b=18\end{cases}.$$

Calculate the sum $S=a^2+b^2$

**A.** 2. **B.** $\sqrt[3]{44}$. **C.** 100 **D.** 10. **E.** $\sqrt{10}$.

**Question 6:** Let b be the square of an odd integer. Find the smallest positive integer $n$ such that $n^3+2n^2=b$,

**Answer:** ____________________

**Question 7:** Calculate the sum of all positive integers which are less than or equal to 114 and not divisible by 7.

**Answer:** ____________________

**Question 8:** Find all integers of three distinct digits such that the sum of all two-digit numbers made up of its 3 digits is equal to it.

**Answer:** ____________________

**Question 9:** Find the value of the expression

$$P = \left(1 - \frac{1}{1+2}\right).\left(1 - \frac{1}{1+2+3}\right)...\left(1 - \frac{1}{1+2+3+...+2017}\right).$$

**Answer:** ____________________

**Question 10:** Find the smallest value of the expression

$$P(x) = x^{2018} + 2x^{2017} + 3x^{2016} + ... + 2018x + 2020$$

**Answer:** ____________________

## Section B.

**Answer the following 5 questions. Present your detailed solution in the space provided.**

**Question 11:** Solve the system of equations

$$\begin{cases} x\left(x+3\right)\left(x+y\right) = 8 \\ x^2 + 4x = 6 - y \end{cases}$$

***Solution:***

**Answer:** ____________________

**Question 12:** Given positive real numbers $x, y, z$ which together satisfy $x^2 + y^2 + z^2 = 3$.

Prove that: $\frac{x}{3 - yz} + \frac{y}{3 - zx} + \frac{z}{3 - xy} \le \frac{3}{2}$.

***Solution:***

**Answer:** ____________________

**Question 13:** Ho, Chi, and Minh participate in the HOMC exam organized by the Hanoi Mathematical Association in 2018. All three have a total score of 207. The greatest common divisor of Ho and Chi's scores is 15 and the greatest common divisor of Ho and Minh's scores is 12. If the smallest common multiple of Chi and Minh scores is maximum, determine the score of each person.

***Solution:***

**Answer:** ____________________

**Question 14:** Find all pairs of integers (x;y) such that

$$\left(x^2+1\right)\left(y^2+1\right)+2\left(x-y\right)\left(1-xy\right)=4xy+9.$$

***Solution:***

**Answer:** ____________________

**Question 15:** Find the area of a right triangle at A with perimeter 72 cm. The difference between the median line and the height derived from A is 7 cm.

***Solution:***

**Answer:** ____________________