

注意：

允許學生個人、非營利性的圖書館或公立學校合理使用本基金會網站所提供之各項試題及其解答。可直接下載而不須申請。

重版、系統地複製或大量重製這些資料的任何部分，必須獲得財團法人臺北市九章數學教育基金會的授權許可。

申請此項授權請電郵 [ccmp@seed.net.tw](mailto:ccmp@seed.net.tw)

**Notice:**

**Individual students, nonprofit libraries, or schools are permitted to make fair use of the papers and its solutions. Republication, systematic copying, or multiple reproduction of any part of this material is permitted only under license from the Chiuchang Mathematics Foundation.**

**Requests for such permission should be made by e-mailing Mr. Wen-Hsien SUN [ccmp@seed.net.tw](mailto:ccmp@seed.net.tw)**

---

# MATHEMATICS EXPLORATION PROBLEMS

---

Name : \_\_\_\_\_

Index Number : \_\_\_\_\_

Country : \_\_\_\_\_



**15<sup>th</sup> International Mathematics and Science Olympiad**

**Zhejiang Province, China**

**1 October 2018**

**Instructions:**

1. Write your name, country and index number on every page of the Answer Sheet.
2. Write your answers only in the Answer Sheet.
3. Answer all questions in Arabic Numerals.
4. There are 6 questions in this paper.
5. Each question is worth 6 marks and partial credit may be awarded.
6. You have 120 minutes to complete this paper.
7. You are provided with some manipulatives for exploration on some questions.
8. Use black pen or blue pen or pencil to write your answer.

---

**Do not turn over this page until you are told to do so.**

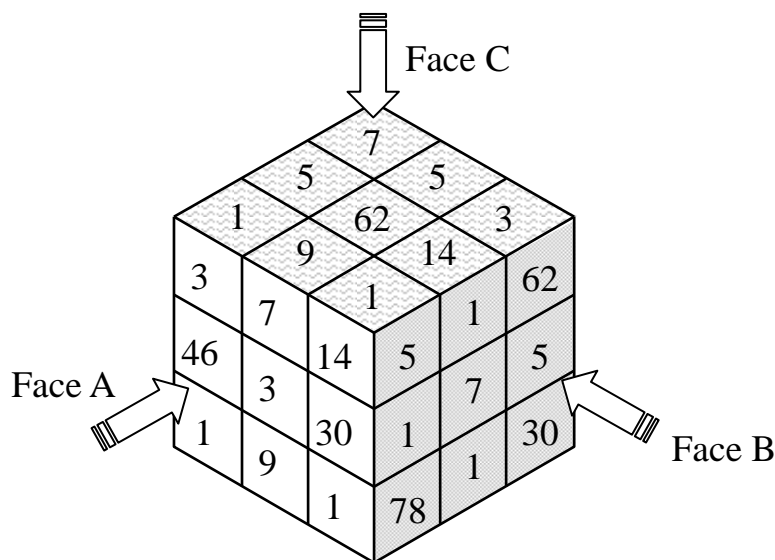
## EXPLORATION PROBLEMS

- (1) Use the digits from 1 to 7 to make the equation below correct. Each letter represents a different one-digit number.

$$\boxed{a} + \boxed{b} \boxed{c} \div \boxed{d} = \boxed{e} \boxed{f}$$

List down all possible equations if  $d > 3$ .

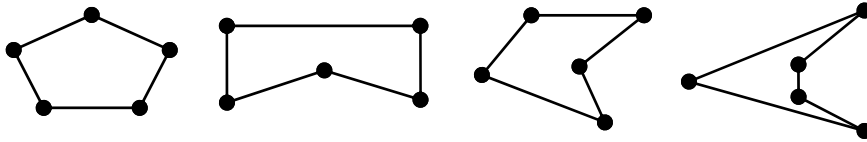
- (2) There are 27 unit cubes, each of size  $1 \times 1 \times 1$ , are glued together to form a larger  $3 \times 3 \times 3$  cube. Each face on each of the  $1 \times 1 \times 1$  unit cubes contains a positive integer such that:
- Two opposite faces of any unit cube must always contain an even number and an odd number. Moreover, the even number must be twice the odd number.
  - When any two unit cubes are placed face-to-face, the two touching faces must always contain an even number and an odd number. Moreover, the odd number must be 1 more than the even number.



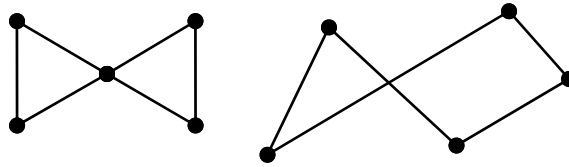
- Place the numbers in the faces of A', B' and C' which are opposite to the faces of A, B, and C, respectively. Find the total sum of the numbers on the faces of A', B' and C'. (2 Marks)
- Find the total sum of the numbers on the faces of all the unit cubes. (4 Marks)

- (3) There are 5 points on the plane, such that no three points lie on one line. Every pair of them is connected by a segment. It is known that it is possible to color five of these segments in red and the rest in blue, so that all segments of each color form a simple pentagon. Draw one example.

The figures below are some examples of a simple pentagon:



The following figures below are NOT simple pentagons (because of self intersections) :



- (4) In a  $4 \times 4$  table, we put a coin on each cell. The coins on some cells are tails up (T), while all the other coins are heads up (H). In each move, you are allowed to change all entries in any row, column or diagonal. Diagonals may be of length 1, 2, 3 or 4. Make the entries in the tables below all H using the required number of moves. Show your steps.

(a) in 4 moves

H	T	H	H
H	H	H	H
T	H	H	H
H	H	H	H

(1 Mark)

(b) in 6 moves

H	T	H	H
H	H	H	H
H	H	H	H
H	H	T	H

(2 Marks)

(c) in 6 moves

H	H	H	H
H	T	H	H
H	H	H	H
H	H	H	H

(3 Marks)

- (5) (a) In a sequence of positive integers, each term after the first term is the sum of its preceding term and the largest digit of that term. What is the largest possible number of successive odd terms in such a sequence? (3 Marks)
- (b) In the following sequence  $\{00, 01, 02, 03, \dots, 39\}$ , the terms were rearranged so that each term after the first is obtained from the preceding one by increasing or decreasing one of its digits by 1. For example, 29 can be followed by 19, 39 or 28, but not by 30 or 20. What is the largest number of terms that can remain in their original places? (3 Marks)
- (6) (a) A chess piece can start anywhere on a  $7 \times 7$  chessboard. It can jump over 4 or 5 vacant squares either vertically or horizontally, but it cannot visit the same square twice. At most how many squares can it visit? (1 Mark)
- (b) A chess piece can start anywhere on a  $9 \times 9$  chessboard. It can jump over 5 or 6 vacant squares either vertically or horizontally, but it cannot visit the same square twice. At most how many squares can it visit? (2 Marks)
- (c) A chess piece can start anywhere on a  $15 \times 15$  chessboard. It can jump over 8 or 9 vacant squares either vertically or horizontally, but it cannot visit the same square twice. At most how many squares can it visit? (3 Marks)