PHÒNG GIÁO DỤC VÀ ĐÀO TẠO
QUẬN LONG BIÊN

ĐỀ CHÍNH THỨC

KỲ THI HỌC SINH GIỎI CẤP QUẬN
GIẢI TOÁN BẰNG TIẾNG ANH HOMC
Năm học 2018-2019
Thời gian làm bài: 120 phút, không kể thời gian giao đề.

Individual Contest - Junior Section
Time limit: 120 minutes

Information:

- You are allowed 120 minutes to complete 10 questions in Section A and 5 questions in Section B .
- Each one of Questions1, 2, 3, 4, and 5 is worth 5 points, and each one of Questions 6, 7, 8, 9, and 10 is worth 10 points. No partial credits are given, and there are no penalties for incorrect answers. Each one of Question 11, 12, 13, 14, and 15 is worth 15 points, and partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Instructions:

- Write down your name, your contestant number and your team's name in the space provided on the first page of the question paper.
- Write your answers on the answer sheets provided.
- For the multiple choice question, stick only the letters (A, B, C, D or E) of your choice .
- You must use either pencil or ball-point pen which is either black or blue.
- The instruments such as protractors, calculators and electronic devices are not allowed to use.
- At the end of the contest you must put the question papers in the envelope provided.

Team: ________________ **Name:** ____________________________

Section A. Multiple choice

Question 1: What is the smallest positive prime factor of the integer $2017^{2019} + 2019^{2017}$?

A. 5 **B.** 7 **C.** 2 **D.** 3 **E.** 8

Question 2: Let p be a real number such that the equation $2y^2 - 8y = p$ has only one solution. Then

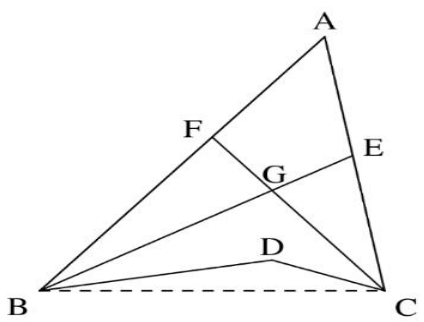
A. $p < 8$ **B.** $p = 8$ **C.** $p > -8$ **D.** $p = -8$ **E.** $p < -8$

Question 3: Which of the following is a possible number of diagonals of a convex polygon?

A. 21 **B.** 32 **C.** 45 **D.** 54 **E.** 63

Question 4: As shown in the diagram, BE and CF bisect ABD and ACD respectively. BE and CF intersect at G. Given that $BDC = 150^0$ and $BGC = 100^0$. Find A in degrees.

A. 60^0. **B.** 50^0. **C.** 45^0. **D.** 55^0. **E.** 75^0



Question 5: What is the largest positive integer n satisfying $n^{200} < 5^{300}$?

A. 9 **B.** 10 **C.** 11 **D.** 12 **E.** 13

Question 6: Sir Jame has a lot of tables and chairs in his house. Each rectangular table seats eight people and each round table seats five people. What is the smallest number of tables he will need to use to seat 35 guests and himself, without any of the seating around these tables remaining unoccupied?

A. 4 **B**. 5 **C.** 6 **D**. 7 **E** .8.

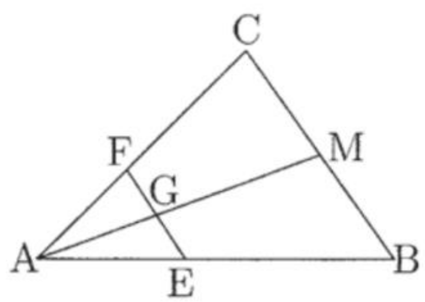
Question 7: A 3-digit number when divided by 57, the remainder is 27; when divided by 217, the remainder is 60. Find the number.

Question 8: The last two digits of $S = 2018^{2018} + 2^{2018}$

Question 9: Given four positive numbers a, b, c, d. Prove that

$$\frac{a}{b+c} + \frac{b}{c+d} + \frac{c}{a+d} + \frac{d}{a+b} \geq 2$$

Question 10: In ΔABC, $AB : AC = 4:3$ and M is the midpoint of BC. E is a point on AB and F is a point on AC such that $AE : AF = 2:1$. It is also given that EF and AM intersect at G with $GF = 72cm$ and $GE = x\,cm$. Find the value of x.
(Note: Figure is not drawn to scale)



Section B. Short questions

Question 11: Let a, b and c be the lengths of the three side of a triangle. Suppose a and b are the roots of the equation. $x^2 + 4(c+2) = (c+4)x$. And the largest angle of the triangle is x^0.

Find the value of x.

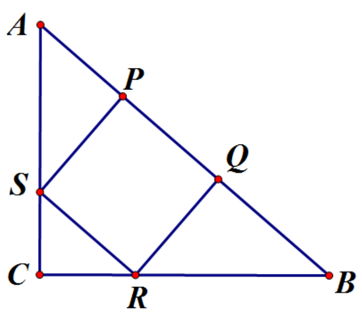
Question 12: How many ordered pairs of integers (x, y) satisfy the equation $x^2 + y^2 = 2(x+y) + xy$

Question 13: ABC is a triangle with sides 3,4 and 5 units. A' is the mirror image of the point A across line BC. Similarly, B' and C' are mirror image of B and C across lines CA and AB. Find the area of triangle $A'B'C'$.

Question 14: Let a, b, c be positive integer such that: $ab + bc = 518$ and $ab - ac = 360$

Find the large possible value of the product abc.

Question 15: In a right angled triangle $ABC, \hat{B} = 41^0$. Square PQRS is inscribed as shown in Figure. Let $AB = c$ and the altitude from C to AB be h. If $\frac{1}{h} + \frac{1}{c} = \frac{2}{3}$, what the length of a side of the square?



— Hết —

Giám thị coi thi không giải thích gì thêm!
Thí sinh nộp lại đề sau khi thi xong.

PHÒNG GIÁO DỤC VÀ ĐÀO TẠO
QUẬN LONG BIÊN

ĐÁP ÁN BÀI THI
GIẢI TOÁN BẰNG TIẾNG ANH HOMC
Năm học 2018-2019

Information:

- You are 10 questions in Section A and 5 questions in Section B .
- Each one of Questions1, 2, 3, 4, and 5 is worth 5 points, and each one of Questions 6, 7, 8, 9, and 10 is worth 10 points. No partial credits are given, and there are no penalties for incorrect answers. Each one of Question 11, 12, 13, 14, and 15 is worth 15 points, and partial credits may be awarded.
- Diagrams shown may not be drawn to scale.

Problem 1. What is the smallest positive prime factor of the integer $2017^{2019} + 2019^{2017}$?

A. 5 B. 7 C. 2 D. 3 E. 8

Answer: C

Problem 2. Let p be a real number such that the equation $2y^2 - 8y = p$ has only one solution. Then

A. $p < 8$ B. $p = 8$ C. $p > -8$ D. $p = -8$ E. $p < -8$

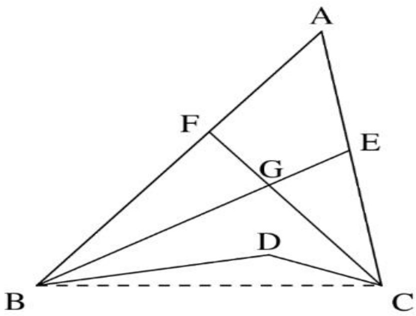
Answer: D

Problem 3. Which of the following is a possible number of diagonals of a convex polygon?

A. 21 B. 32 C.45 D.54 E. 63

Answer: D.

Problem 4: As shown in the diagram, BE and CF bisect ABD and ACD respectively. BE and CF intersect at G. Given that $BDC = 150^0$ and $BGC = 100^0$. Find A in degrees.



A. 60^0. **B.** 50^0. **C.** 45^0. **D.** 55^0.
E. 75^0
Answer:B

Problem 5. What is the largest positive integer n satisfying $n^{200} < 5^{300}$?

A. 9 B. 10 C. 11 D. 12 E. 13

Answer: C

Problem 6. Sir Jame has a lot of tables and chairs in his house. Each rectangular table seats eight people and each round table seats five people. What is the smallest number of tables he will need to use to seat 35 guests and himself, without any of the seating around these tables remaining unoccupied?

A. 4 **B.** 5 **C.** 6 **D.** 7 **E** .8.

Answer: C

Problem 7. A 3-digit number when divided by 57, the remainder is 27; when divided by 217, the remainder is 60. Find the number.

Answer: 711

Problem 8. The last two digits of $S = 2018^{2018} + 2^{2018}$ is?

Solution: We have

* $2018 \equiv 18 (mod\, 100) \Rightarrow 2018^2 \equiv 18^2 (mod\, 100)$

We also have $18^2 \equiv 24 (mod\, 100)$. Therefore, we have:

$2018^2 \equiv 24 (mod\, 100) (1) \Rightarrow 2018^4 \equiv 24^2 (mod\, 100)$,

and $24^2 \equiv 76\ (mod\, 100)$.

Hence, $2018^4 \equiv 76\ (mod\, 100) \Rightarrow (2018^4)^{504} \equiv 76 (mod\, 100)$ (2)

Combining (1) and (2),
$2018^{2018} = (2018^4)^{504}.2018^2 \equiv 76.24 (mod\, 100) \equiv 24 (mod\, 100)$.

* We have $2^{2018} = (2^{10})^{200}.2^{18} \equiv (-1)^{200}.2^{18} (mod\, 25) \equiv 44 (mod\, 25)$

$\Rightarrow 2^{2018} = 25k + 44 (k \in N)$

We also have $2^{2018} \vdots 4 \Rightarrow k \vdots 4$ because $(25, 4) = 1$

Hence, $2^{2018} = 100k + 44$ (2)

Combining, we the number 68 subjects to the question.

Answer: 68.

Problem 9. Given four positive numbers a, b, c, d. Prove that $\frac{a}{b+c} + \frac{b}{c+d} + \frac{c}{a+d} + \frac{d}{a+b} \geq 2$

Solution: Apply extra inequality $\frac{1}{xy} \geq \frac{1}{(x+y)^2}$ $(x, y > 0)$

We have: $\frac{a}{b+c} + \frac{c}{d+a} = \frac{a(d+a) + c(b+c)}{(b+c)(d+a)} \geq 4\frac{a^2 + c^2 + ad + bc}{(a+b+c+d)^2}$ (1)

Similary, $\frac{b}{c+d} + \frac{d}{a+b} \geq 4\frac{b^2 + d^2 + ab + cd}{(a+b+c+d)^2}$ (2)

Phus (1) to (2), we have: $\frac{a}{b+c} + \frac{b}{c+d} + \frac{c}{a+d} + \frac{d}{a+b} \geq 4\frac{a^2 + b^2 + c^2 + d^2 + ad + bc + ab + cd}{(a+b+c+d)^2}$

Then we prove

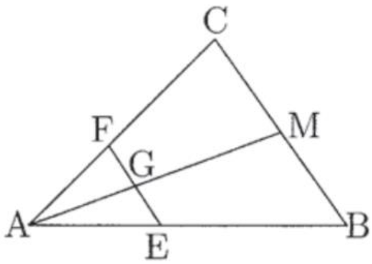
$$4\frac{a^2+b^2+c^2+d^2+ad+bc+ab+cd}{(a+b+c+d)^2} \geq 2$$

$\Leftrightarrow\ 4(a^2+b^2+c^2+d^2+ad+bc+ab+cd) \geq 2(a+b+c+d)^2$

$\Leftrightarrow\ 2a^2+2b^2+2c^2+2d^2-4ac-4bd \geq 0$

$\Leftrightarrow\ (a-c)^2+(b-d)^2 \geq 0$

Answer: $x=673,\ y=672,\ z=671$



Problem 10. In ΔABC, $AB:AC=4:3$ and M is the midpoint of BC. E is a point on AB and F is a point on AC such that $AE:AF=2:1$. It is also given that EF and AM intersect at G with $GF=72cm$ and $GE=x\ cm$. Find the value of x.

Answer: 108

Problem 11. Let a, b and c be the lengths of the three side of a triangle. Suppose a and b are the roots of the equation.

$$x^2+4(c+2)=(c+4)x$$

And the largest angle of the triangle is x^0. Find the value of x.

Answer: 90

Solution:

Since a,b are the roots of the equation $x^2-(c+4)x+4(c+2)=0$, it follows that

$a+b=c+4$ and $ab=4(c+2)$

Then $a^2+b^2=(a+b)^2-2ab=(c+4)^2-8(c+2) \Leftrightarrow a^2+b^2=c^2$. Hence the triangle is right-angled, and $x=90$.

Problem 12. How many ordered pairs of integers (x,y) satisfy the equation

$$x^2+y^2=2(x+y)+xy$$

Answer: 6

Problem 13. Let line AA' intersect BC at H, $B'C'$ at H'. Then $A'H \perp BC, AH' \perp B'C'$. Also in lengths,

$B'C' = BC,\ A'H' = 3AH.$

So the area of $\Delta A'B'C'$ is 3 times the area of ΔABC, which is 3 × 6 =18.

Answer: 18

Problem 14. Let a,b,c be positive integer such that: $ab+bc=518$ and $ab-ac=360$

Find the large possible value of the product abc.

Answer: ***1008***

Solution: $bc + ac = 518 - 360 = 158 \Rightarrow c(a+b) = 2.79$. Thus c must be $1, 2$ or 79.

If $c = 79$ then $a + b = 2 \Rightarrow a = b = 1, c = 79$, which not satisfy the given equations.

If $c = 2$ then $a + b = 79$. Substitute these values into the second equation, we get:

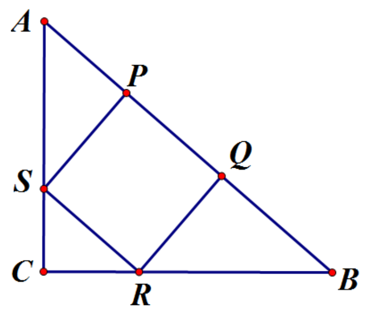
$a(79 - a) - 2a = 360 => a^2 - 77a + 360 = 0 => a = 5 \text{ or } 72$.

When $a = 5, b = 74, c = 2$ have $abc = 740$.

When $a = 72, b = 7, c = 2$ we have $abc = 1008$

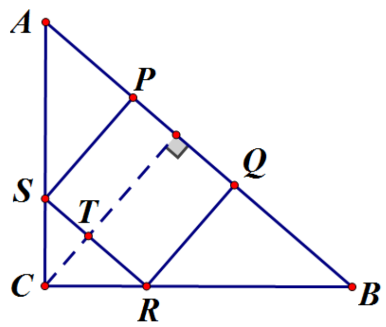
If $c = 1$ proceeding as before, we get $a^2 - 157a + 360 = 0$ which has no integer solution for a

Thus the largest possible value of $abc = 1008$



Problem 15. In a right angled triangle $ABC, \hat{B} = 41^0$. Square PQRS is inscribed as shown in Figure. Let $AB = c$ and the altitude from C to AB be h. If $\frac{1}{h} + \frac{1}{c} = \frac{2}{3}$, what the length of a side of the square?

Answer: $\frac{3}{2}$



Solution: In figure, we draw the altitude from C to AB and label its intersection with SR by T as shown. Let $SP = SR = x$. Then $CT = h - x$. Observe that $\Delta SRC \backsim \Delta ABC$. Thus, the corresponding side are in the same ratio:

$$\frac{CT}{SR} = \frac{h}{c} \Rightarrow \frac{h - x}{x} = \frac{h}{c} \Rightarrow x = \frac{ch}{c + h} = \frac{1}{\frac{1}{c} + \frac{1}{h}} = \frac{3}{2}$$

Học sinh giải cách khác đúng vẫn cho điểm tối đa.

Tổ giám khảo thống nhất điểm thành phần của các ý nhưng không thay đổi tổng điểm mỗi câu.

Điểm làm bài làm tròn đến 1 chữ số thập phân.