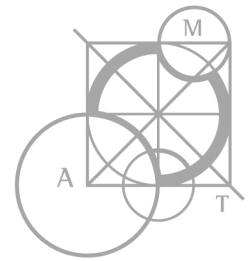


## Australian Intermediate Mathematics Olympiad 2018

### Questions

1. Let  $x$  denote a single digit. The tens digit in the product of  $2x7$  and  $39$  is  $9$ . Find  $x$ .  
[2 marks]
2. If  $234_{b+1} - 234_{b-1} = 70_{10}$ , what is  $234_b$  in base  $10$ ?  
[3 marks]
3. The circumcircle of a square  $ABCD$  has radius  $10$ . A semicircle is drawn on  $AB$  outside the square. Find the area of the region inside the semicircle but outside the circumcircle.  
[3 marks]
4. Find the last non-zero digit of  $50! = 1 \times 2 \times 3 \times \dots \times 50$ .  
[3 marks]
5. Each edge of a cube is marked with its trisection points. Each vertex  $v$  of the cube is cut off by a plane that passes through the three trisection points closest to  $v$ . The resulting polyhedron has  $24$  vertices. How many diagonals joining pairs of these vertices lie entirely inside the polyhedron?  
[3 marks]
6. Let  $ABCD$  be a parallelogram. Point  $P$  is on  $AB$  produced such that  $DP$  bisects  $BC$  at  $N$ . Point  $Q$  is on  $BA$  produced such that  $CQ$  bisects  $AD$  at  $M$ . Lines  $DP$  and  $CQ$  meet at  $O$ . If the area of parallelogram  $ABCD$  is  $192$ , find the area of triangle  $POQ$ .  
[4 marks]

PLEASE TURN OVER THE PAGE FOR QUESTIONS 7, 8, 9 AND 10



7. Two different positive integers  $a$  and  $b$  satisfy the equation  $a^2 - b^2 = 2018 - 2a$ .  
What is the value of  $a + b$ ?

[4 marks]

8. The area of triangle  $ABC$  is 300. In triangle  $ABC$ ,  $Q$  is the midpoint of  $BC$ ,  $P$  is a point on  $AC$  between  $C$  and  $A$  such that  $CP = 3PA$ ,  $R$  is a point on side  $AB$  such that the area of  $\triangle PQR$  is twice the area of  $\triangle RBQ$ . Find the area of  $\triangle PQR$ .

[4 marks]

9. Prove that 38 is the largest even integer that is *not* the sum of two positive odd composite numbers.

[4 marks]

10. A pair of positive integers is called *compatible* if one of the numbers equals the sum of all digits in the pair and the other number equals the product of all digits in the pair. Find all pairs of positive compatible numbers less than 100.

[5 marks]

*Investigation*

Find all pairs of positive compatible numbers less than 1000 with at least one number greater than 99.

[3 bonus marks]